EE649 Speech Processing by Computer
Linear Predictive Coding (LPC) Summary

Assume: signal $\{s_n\}$

windowed signal $s'_n = s_n w_n, \quad$ for $0 \le n < N$

I. Time Domain Formulation

$$H(z) = \frac{1}{1 - \sum_{i=1}^{p} a_i z^{-i}} \qquad \text{vocal tract filter}$$

$$s_n = \sum_{i=1}^{p} a_i \, s_{n-i} + x_n \qquad \text{signal}$$

$$e_n = s_n - \sum_{i=1}^{p} a_i \, s_{n-i} \qquad \text{residual signal (error)}$$

a. Autocorrelation method on windowed signal $\{s'\}$

Define: $$R_N(i) = \sum_{n=0}^{N-1-i} s'_n \, s'_{n+i} \qquad \text{for } 0 \le i \le p$$

Solve: $$\sum_{i=1}^{p} a_i \, R_N(|k-i|) = R_N(k) \qquad \text{for } 1 \le k \le p$$

$$E_{min}^{AC} = R_N(0) - \sum_{i=1}^{p} a_i R_N(i)$$

b. Covariance method

Define: $$\phi_N(i,k) = \sum_{n=0}^{N-1} s_{n-i} s_{n-k} \qquad \text{for } 0 \le i, \; k \le p$$

Solve: $$\sum_{i=1}^{p} a_i \phi_N(i,k) = \phi_N(k,0) \qquad \text{for } 1 \le k \le p$$

$$E_{min}^{cov} = \phi_N(0,0) - \sum_{i=1}^{p} a_i \phi_N(i,0)$$

II. Frequency Domain Formulation (Inverse Filtering)

$$A(z) = \frac{1}{H(z)} = 1 + \sum_{i=1}^{p} b_i z^{-i} \qquad \text{inverse filter}$$

$$H(z) = \frac{1}{1 + \sum_{i=1}^{p} b_i z^{-i}} \qquad \text{vocal tract filter}$$

Solve: $$\sum_{i=1}^{p} b_i R_N(|k-i|) = -R_N(k) \qquad \text{for } 1 \le k \le p$$

$$E_{min}^{IF} = R_N(0) + \sum_{i=1}^{p} b_i R_N(i)$$

$$e_n = s_n + \sum_{i=1}^{p} b_i s_{n-i} \qquad \text{"residual signal"}$$

The autocorrelation and inverse filtering formulations are equivalent if

$$b_i = -a_i \text{ for } 1 \le i \le p$$